Non-Circular Pipe Friction

The frictional head loss in circular pipes is usually calculated by using the Darcy-Weisbach formula with a Darcy Friction factor. For circular pipes the inner pipe diameter is used is used to calculate the Reynolds number and to calculate the relative roughness of the pipe, which are both used to calculate the Darcy Friction factor.

To calculate the frictional head loss non-circular pipes the method must be adapted to use the Hydraulic Diameter instead of the internal dimensions of the pipe.

Hydraulic Diameter = 4 x cross sectional area of flow / wetted perimeter

For a round pipe the $D_h = 4 \times (\pi \times d^2 / 4) / (\pi \times d) = d$

For a rectangular duct the $D_h = 4 \times (w \times h) / 2 \times (w + h)$ where w = width, h = height

For an elliptical duct the $D_h = 4 \times (\pi \times a \times b) / \pi \times \sqrt{} [(2 \times (a^2 + b^2)) – ((a - b)^2/2)]$

where a = major diameter / 2, b = minor diameter /2 ,

Note: the formula uses an approximation for the circumference of an elliptical duct.

For an annulus formed by placing a smaller diameter pipe inside a larger diameter pipe the cross sectional area of flow will be the cross sectional area of the larger pipe calculated using the inner pipe diameter minus the cross sectional area of the smaller pipe calculated using the outer pipe diameter. The wetted perimeter will be the inner circumference of the larger pipe plus the outer circumference of the smaller pipe.

$D_h = 4 \times (\pi \times (d1^2 – d2^2) / 4) / (\pi \times (d1 + d2))$

where d1 = inner diameter of larger pipe, d2 = outer diameter of smaller pipe

Example calculation of pipe friction factors:

1. Round pipe:

A round steel pipe 0.4 m internal diameter x 10.0 m long carries a water flow rate of 349.1 litres/sec (20.946 m^3/min). The temperature of the water is 10^o C (50^o F).

D_h = Internal diameter of pipe = 0.4 m

Pipe cross sectional area = $\pi \times 0.400^2/4 = 0.1256$ m^2

Flow velocity = 20.94/0.1256/60 = 2.778 m/s

Relative roughness = 0.000046/0.4 = 0.000115

Re = v x D_h / (kinematic viscosity in m^2/s) = 2.778 x 0.4 / 0.000001307 = 850191

Friction factor = 0.014 (plotted from Moody chart)

$h_f = f\,(L / D_h) \times (v^2 / 2g) = 0.014 \times (10 / 0.4) \times (2.778^2 / (2 \times 9.81)) = 0.138$ m head

where:

h_f = frictional head loss (m)

f = friction factor

L = length of pipe work (m)

D_h = Hydraulic diameter (m)

v = velocity of fluid (m/s)

g = acceleration due to gravity (m/s ²)

2. Rectangular duct:

A rectangular steel duct 0.6 m wide x 0.3 m high x 10.0 m long carries a water flow rate of 500 litres/sec (30 m^3/min). The temperature of the water is 10^o C (50^o F).

D_h = 4 x (0.6 x 0.3) / 2 x (0.6 + 0.3) = 0.4 m
Duct cross sectional area = 0.6 x 0.3 = 0.18 m^2
Flow velocity = 30.00/0.18/60 = 2.778 m/s
Relative roughness = 0.000046/0.4 = 0.000115
Re = v x D_h / (kinematic viscosity in m^2/s) = 2.778 x 0.4 / 0.000001307 = 850191
Friction factor = 0.014 (plotted from Moody chart)

$h_f = f\ (L / D_h) \times (v^2 / 2g) = 0.014 \times (10 / 0.4) \times (2.778^2 / (2 \times 9.81)) = 0.1377$ m head

where:
h_f = frictional head loss (m)
f = friction factor
L = length of pipe work (m)
D_h = Hydraulic diameter (m)
v = velocity of fluid (m/s)
g = acceleration due to gravity (m/s ^{2})

Pseudo check calculation: A steel pipe with an internal diameter of 0.400 m x 10 m long carrying a water flow rate of 349.1 litres/sec (20.946 m^3/min) will have the same flow velocity as the rectangular duct. If the water temperature is 10^o C (50^o F) the calculated frictional pressure drop through the steel pipe is 0.138 m head.

3. Elliptical duct:

An elliptical duct made from aluminium has internal dimensions of 0.8 m at its widest point and 0.3 m at is highest point. The duct is 10.0 m long and carries a water flow rate of 400 litres/sec (24 m^3/min). The temperature of the water is 10^o C (50^o F).

a = major diameter / 2 = 0.800 / 2 = 0.400
b = minor diameter / 2 = 0.300 / 2 = 0.150
Duct cross sectional area = π x a x b = π x 0.400 x 0.150 = 0.1885 m^2
Duct circumference = $\pi \times \sqrt{}\ [(2 \times (a^2 + b^2)) - ((a - b)^2/2)]$
$= \pi \times \sqrt{}\ [(2 \times (0.4^2 + 0.15^2)) - ((0.4 - 0.15)^2/2)] = \pi \times \sqrt{}[0.365 - 0.03125] = 1.8149$ m
D_h = 4 x 0.1885 / 1.8149 = 0.415 m
Flow velocity = 24.00 / 0.1885 / 60 = 2.1220 m/s
Relative roughness = 0.0000015 / 0.415= 0.000003615
Re = v x D_h / (kinematic viscosity in m^2/s) = 2.1220 x 0.415 / 0.000001307 = 673780
Friction factor = 0.0123 (plotted from Moody chart)

$h_f = f\ (L / D_h) \times (v^2 / 2g) = 0.0123 \times (10 / 0.415) \times (2.1220^2 / (2 \times 9.81)) = 0.068$ m head

where:
h_f = frictional head loss (m)
f = friction factor
L = length of pipe work (m)
D_h = Hydraulic diameter (m)
v = velocity of fluid (m/s)
g = acceleration due to gravity (m/s ^{2})

Pseudo check calculation: An aluminium pipe with an internal diameter of 0.415 m x 10 m long carrying a water flow rate of 287.1 litres/sec (17.226 m^3/min) will have the same flow velocity as the elliptical duct. If the water temperature is 10^o C (50^o F) the calculated frictional pressure drop is 0.069 m head.

4. Annulus:

An annulus section is formed by placing a stainless steel pipe with an outer diameter of 350 mm inside a stainless steel pipe with an inner diameter of 600. The annulus section is 10 m long and carries a water flow rate of 600 litres/sec (36.00 m^3/min). The water temperature is 20^o C (68^o F).

Inner cross sectional area of the larger pipe = π x 0.600^2 / 4 = 0.2827 m^2
Outer cross sectional area of the smaller pipe = π x 0.350^2 / 4 = 0.0962 m^2
Cross sectional area of the annulus = 0.2827 - 0.0962 = 0.1865 m^2

Inner circumference of the larger pipe = π x 0.600 = 1.8850 m
Outer circumference of the smaller pipe = π x 0.350 = 1.0995 m
Wetted perimeter = 1.8850 + 1.0995 = 2.9845 m

D_h = 4 x 0.1865 / 2.9845 = 0.250 m
Flow velocity = 36.00 / 0.1865 / 60 = 3.217 m/s
Relative roughness = 0.000045 / 0.250 = 0.000180
Re = v x D_h / (kinematic viscosity in m^2/s) = 3.217 x 0.250 / 0.000001004 = 801045
Friction factor = 0.0146 (plotted from Moody chart)

$h_f = f\ (L / D_h) \times (v^2 / 2g) = 0.0146 \times (10 / 0.250) \times (3.217^2 / (2 \times 9.81)) = 0.307$ m head

where:
h_f = frictional head loss (m)
f = friction factor
L = length of pipe work (m)
D_h = Hydraulic diameter (m)
v = velocity of fluid (m/s)
g = acceleration due to gravity (m/s^2)

Pseudo check calculation: A stainless steel pipe with an internal diameter of 0.250 m x 10 m long carrying a water flow rate of 157.917 litres/sec (9.475 m^3/min) will have the same flow velocity as the annulus. If the water temperature is 20^o C (68^o F) the calculated frictional pressure drop through the steel pipe is 0.307 m head.